

Effect of variable mechanical resistance on electrodynamic alternator efficiency



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ARTICLE INFO

Article history:

Received 23 March 2014

Accepted 7 September 2014

Keywords:

Electroacoustic transduction efficiency
Thermoacoustics
Electrodynamic alternator
Mechanical resistance
Transfer impedance

ABSTRACT

The rapidly growing energy market constantly discovers new alternatives for generating environmentally-friendly electricity from sustainable energy sources. An externally-heated traveling wave thermoacoustic Stirling heat engine is a leading candidate, operating using a variety of viable heat sources. Although this engine holds great promise for a low-cost and maintenance-free solution, its current reported efficiency still inhibits its market penetration, probably owing to the friction introduced by the single moving element – the linear alternator.

This research quantifies the main parameters affecting the complex thermal–acoustical–electrical system, clarifying the critical role of frictional losses. An analytical model has been developed, enabling examination of the influence of the critical physical parameters on the electro-acoustic conversion efficiency. A measurement method for precise determination of the mechanical friction constant has been developed, which enables measuring the friction at the engine's working frequency. A direct measurement of an engine's transfer impedance at room temperature enables to find the exact natural frequency before field operation, and calibrate external hardware accordingly. A detailed simulation using DeltaEC™ indicates that the tight seal gap between the moving piston and its cylinder has a significant impact on the system's overall efficiency.

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1. Introduction

Efficient Thermo Acoustic Stirling heat Engines (TASE), allowing for reliable operation without moving parts or sliding seals, have recently stirred a revival of interest. These environmentally-friendly, externally-heated engines may operate using a variety of viable heat sources, including natural gas, biofuels, open fire or solar radiation. Additionally, the lack of need for complex fabrication processes in the simple TASE structure makes it an excellent candidate for commercial solar electricity production. Nowadays, the rapidly growing solar-thermal market finds great promise in new methods for converting Concentrated Solar Power (CSP) into electricity.

The “traveling wave” thermo-acoustic engine employs the inherently reversible Stirling cycle, theoretically capable of attaining Carnot efficiency. A practical demonstration of a thermoacoustic engine has produced a thermal efficiency (heat to acoustic power) of 30% [1] at 725 °C, comparable to that of modern internal combustion engines [2] (25–40% heat to mechanical

power). Converting acoustic power into mechanical and/or electrical power is no small challenge, and forms the subject of this work. A commercial thermoacoustic system has produced over 17 kW of acoustic power [3] which was then used to liquefy natural gas (Methane liquefies at 115 K at atmospheric pressure) at a rate as high as 140 gpd. In 2004, a remarkable thermoacoustic engine, converting heat into electricity, was developed by Northrop Grumman Space and Technology group with Los Alamos National Laboratory under a NASA contract [4]. This engine produced 39 W of electrical power at 18% overall conversion efficiency (heat to electricity), where a pair of dual opposed linear alternators were driven by acoustic power [5]. Using Inconel 625 for the hot heat exchanger has allowed working temperatures as high as 650 °C which enabled the first implementation of a traveling wave thermoacoustic engine for efficient electricity production.

Higher power levels have been demonstrated in 2005 by Luo et al. [6], as the nonlinear dissipation losses were reduced in the acoustic resonator by employing a tapered resonance tube. In 2008 [7] more than 100 W electrical output power were produced through proper coupling between the linear alternator and the thermoacoustic engine, and in 2011 [8] the electrical output power was further increased to as much as 481 W with thermal to

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Nomenclature

A	cross section area, m^2	Z_m^e, Z_a^e	electrical equivalent of mechanical, acoustic impedance, Ω
B	magnetic field, T	Z_c	transfer impedance, A-s/m
(Bl)	alternator constant, N/A	$Z_m, Z_{m,a}$	mechanical impedance (total, mechanical–acoustical), N-s/m
C	compliance, m^3/Pa	γ	c_p/c_v adiabatic ratio
C_e	capacitance, F	η	efficiency
F	force, N	ξ	displacement, m
f	frequency, Hz	ρ, ρ_m, ρ_1	total, mean part (real), oscillatory part (complex) of density kg/m^3
I, I_1	total (real), oscillatory part (complex), of electrical current, A	$\phi_{p_1, U_1}, \phi_{\Delta p_1, U_1}$	phase between p_1 or Δp_1 and U_1
I_a	acoustic intensity, W/m^2	ω	angular frequency 1/s
$K, K_{\text{gas-spring}}, K_{\text{eff}}$	spring constant (mechanical), gas spring, effective, N/m	Supplementary notation	
L	inertance, kg/m^4	$\Re\{\}$	real part
L_e	electrical inductance, H	$\Im\{\}$	imaginary part
l	characteristic or effective length, m	$\ $	absolute value, Norm
m	mass, kg	$()^*$	complex conjugate
p, p_m, p_1	total, mean part (real), oscillatory part (complex) of pressure Pa, (N/m^2)	i	$\sqrt{-1}$
$\vec{P}$	Power, W	$\langle \rangle$	time average
R, R_e, R_L	electrical resistance (internal, external load), Ω	Δ	difference
R_m	mechanical damping, N-s/m	Subscript	
R_f	acoustic flow resistor, N-s- m^3	AE	acoustic-electrical
$\vec{r}$	space vector, m	ALT	alternator
T, T_m, T_1	total, mean part (real), oscillatory part (complex) of temperature, K	a	acoustic
t	time, s	c	characteristic
U, U_1	total (real), oscillatory part (complex), of volumetric flow rate, m^3/s	CS	compression space
u, u_1	total (real), oscillatory part (complex), x component of velocity, m/s	EMF	electromotive force
V	volume, m^3	e	electric
$V_{\text{EMF}}, V_1, V_{\text{int}}$	voltage, V	ext	external
$\vec{v}$	velocity vector, m/s	EA	electro-acoustic
x	axial coordinate, m	int	internal
y, z	transverse coordinate, m	m	mechanical
Z_a, Z_{ALT}	acoustic impedance (total, acoustic representation of alternator impedance), N-s/ m^5	m, a	mechano-acoustic
$Z_{e,\text{int}}, Z_{e,\text{ext}}, Z_{e,\text{total}}$	electrical (internal, external, total) impedance, Ω	REG	regenerator
		TA	thermoacoustic
		TASE	Thermo Acoustic Stirling Engine
		1	oscillatory variable

electricity efficiency of 12.65% at 650 °C, using 3.5 MPa pressurized helium at typical engine working conditions.

Additional recent projects include the work done at the Energy Research Center of the Netherlands (ECN) by Tijani and co-workers [9–11]; the FP7 cooperative project on thermoacoustic technology for energy applications led by Spoelstra, Thermeau, Jaworski and de Blok [12–14]; the power converter built by Telesz at Georgia Institute of Technology [15]; the large-scale-engine research performed by Qiu et al. at the Institute of Refrigeration and Cryogenics, Zhejiang University, Hangzhou, China [16]; the investigation of low cost regenerator materials conducted by Yu, Jaworski and Abduljalil at the School of Mechanical, Aerospace and Civil Engineering, University of Manchester, England [17,18]; and the Score-Stove™ project aiming to generate electricity in developing countries using thermo-acoustics powered by burning wood. This project initiated a broad collaboration between the University of Nottingham, University of Manchester, Queen Mary University of London (QMUL), Imperial College London and Los Alamos National Laboratory (LANL) [19–23]; Recently, a solar-operated thermoacoustic Stirling engine has been reported for the first time to produce as much as 200 W of electrical power with overall efficiency of 12.7% (heat to electricity) at 750 °C [24]. The additional option of heating the engine with solar-thermal energy has led to

a compromise of the performance of this prototype compared to earlier versions [8].

Despite its conceptual simplicity and high reliability, nowadays further efficiency improvements are necessary for a complete TASE-based system to gain a market share. Other well-established heat-to-electricity conversion systems have been around for a long time, and competing with them, mainly for efficiency, represents an important challenge for the TASE system.

The main objective of this research has been to provide a qualitative understanding as well as quantitative analysis of the key parameters affecting the complex thermal–acoustical–electrical system. A fairly involved trade-off exists between different parameters affecting the performance of the system. In particular, we address the effect of mechanical friction, known to have a deleterious impact on the system performance. To better understand this effect, an analytical model is developed in this paper. Its results will be compared to a detailed thermoacoustic simulation using DeltaEC™ in a separate paper. An alternative method of measurement for precise determination of the mechanical friction constant was developed and is described.

Section 2 gives a short background of thermo-acoustic-electrical transduction. An analytical model of the involved processes is presented in Section 3. Section 4 describes the experimental setup

and main experimental results, followed by a short summary and main conclusion in Section 5.

2. Thermo-acoustic-electrical transduction basics

Thermoacoustic oscillations are the outcome of interactions between heat and sound. The development of thermoacoustic technology has progressed with many offshoots and branches. The first commercial product based on the thermoacoustic effect became available in the market approximately 200 years after its recognition. In parallel, Stirling cycle engines were investigated for almost 150 years, until they found their use in solar powered systems, demonstrating the current world record for solar to grid efficiency-32% [25], exceeding the previous one – 31.25% [26], which can be compared to the highest theoretical efficiency of a single p–n junction solar cell – 33.7% [27].

The principal idea in a TASE is based on a traveling wave passing through the gas in a regenerative heat exchanger (regenerator) in a sequence of processes: displacement toward a high (heat source) temperature, depressurization, displacement toward a low (heat sink) temperature, and pressurization. The gas experiences thermal expansion during the displacement toward the higher temperature and thermal contraction during the displacement toward the lower temperature, hence the acoustic power is amplified during the cycle; this is the origin of the available-work of the traveling-wave engine. The overall efficiency η for electricity generation from heat depends on the thermoacoustic efficiency η_{TA} as well as the electro acoustic transduction efficiency η_{EA} through their product

$$\eta = \eta_{TA} \cdot \eta_{EA} \quad (2.1)$$

Wakeland [28] discussed the complexity in reaching the highest overall efficiency in thermoacoustic systems by correct impedance matching, while Paek et al. [29] as well as Yu et al. [22] suggested several experimental verification techniques. In most cases, the optimal design is found through some compromise between maximum output power and overall efficiency. Usually, some degree of freedom exists in the integration of a given alternator and a thermoacoustic engine.

The notations used herein below follow closely the ones defined by Swift [30,31]. A lossless acoustic wave having an angular frequency ω , related to the primary frequency f through

$$\omega = 2\pi f \quad (2.2)$$

may be described by the following two governing equations – the continuity equation

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{v}) \quad (2.3)$$

and the inviscid momentum equation

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} \right) = -\vec{\nabla} p \quad (2.4)$$

Using the “Rott lossless approximation” [32–34] means that all time-dependent variables are small compared to their mean value and only a single frequency is considered (no higher harmonics):

$$p(\vec{r}, t) = p_m + \Re\{p_1(x)e^{i\omega t}\} \quad (2.5)$$

$$\rho(\vec{r}, t) = \rho_m(x) + \Re\{\rho_1(\vec{r})e^{i\omega t}\} \quad (2.6)$$

$$T(\vec{r}, t) = T_m(x) + \Re\{T_1(\vec{r})e^{i\omega t}\} \quad (2.7)$$

$$U(x, t) = \Re\{U_1(x)e^{i\omega t}\} \quad (2.8)$$

In this approximation the mean pressure p_m has no spatial distribution whereas the pressure gradients of the oscillating part p_1 are allowed only in the x direction. The mean temperature T_m ,

as well as the mean density ρ_m , are assumed constant in each cross section while only the oscillatory parts T_1 and ρ_1 can vary in the y and z directions. The volumetric flow rate U has only an oscillatory part U_1 since no large mean flow rate is expected. The time independent portion of U , referred to as streaming, is only second order small, hence would not be discussed here.

Using this approximation for an ideal gas, the continuity equation yields [30]

$$p_1 = -\frac{\gamma p_m}{i\omega A \Delta x} \Delta U_1 = \frac{i}{\omega C} \Delta U_1 \quad (2.9)$$

and the momentum equation results in

$$\Delta p_1 = -i\omega \frac{\rho_m \Delta x}{A} U_1 = -i\omega L U_1 \quad (2.10)$$

while the pressure is related to the volumetric flow rate through

$$p_1 = -Z_a U_1 \quad (2.11)$$

with the definitions of compliance C and inertance L accordingly:

$$\text{Compliance} - C = \frac{V}{\gamma p_m} \quad (2.12)$$

$$\text{Inertance} - L = \frac{\rho_m \Delta x}{A} \quad (2.13)$$

Considering a given geometry, the acoustic impedance Z_a could be specified using combinations of compliances, inertances and resistive elements, as will be explained next. These relations become very useful in using the analogy to a lumped electrical circuit.

An electricity-producing thermoacoustic engine may generally be presented according to Fig. 1(a) [4,5]. The acoustic elements are the compliance C , compression space CS , the inertance L and the regenerator REG . Fig. 1(b) describes the elements of the alternator, and its connection to the TASE loop. The two pistons are each driven by a magnet and inductor, and held by a spring. A LVDT is shown for measurement purposes only – to determine the location of the piston during operation.

3. Analytical model

Fig. 2 describes a simplified acoustical representation for the system of Fig. 1 in terms of an electrical analog, where the regenerator REG may be represented as a combination of an acoustic flow amplifier and a flow resistor R_f [35]. The acoustic representation of the alternator impedance Z_{ALT} includes a contribution from two terms: the alternator mechanical impedance as well as electrical impedance determined by the alternator's configuration and its main elements. Normally the electrical portion is affected by external hardware such as resistive load and tuning capacitor, as well as the alternator's internal structure elements – the magnetic field strength, the winding-coil length and diameter, the flux closure, and its relative to its equilibrium position. The mechanical effect is usually determined by the piston's cross sectional area, its moving mass, the mechanical friction and the alternator spring-constant. The exact form and all related details will be elaborated later in this paper.

Three perspectives may be used to analyze the complex system response, each having its pros and cons: Fig. 2 shows these perspectives, described next, as colored arrows. The *TA engine analysis mode* includes all physical phenomena though it typically requires a very complex analysis for a complete inspection; the *speaker analysis mode* is directly available using a simple set of experimental test procedures though it uses the system in (an almost) opposite course of operation; while the *EA analysis mode* may be used to draw exact conclusions regarding the system performance; nevertheless, it suffers some ambiguity concerning the incoming

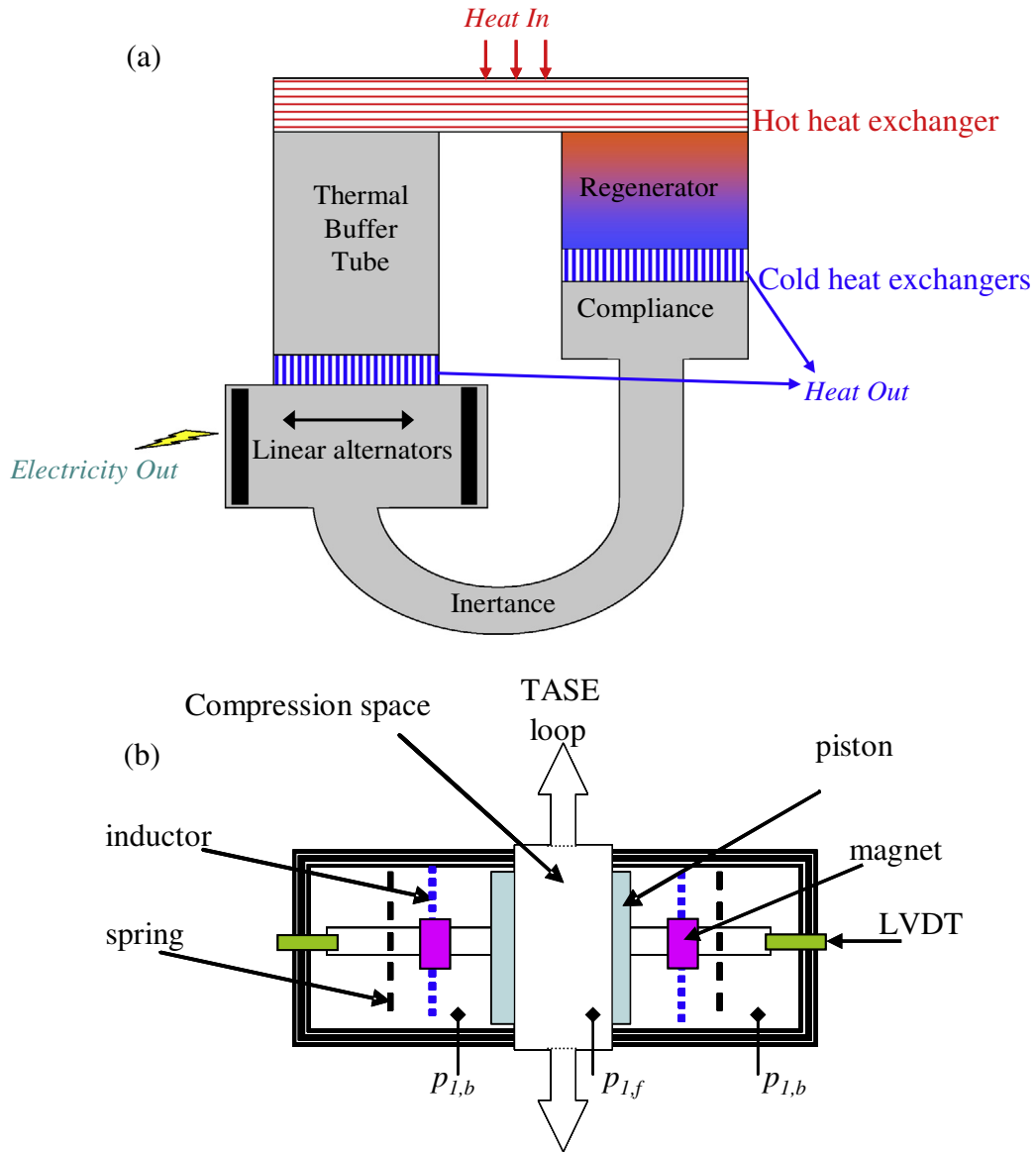


Fig. 1. Schematic representation of a Thermo-Acoustic Stirling engine: (a) complete TASE and (b) alternator in its pressure vessel.

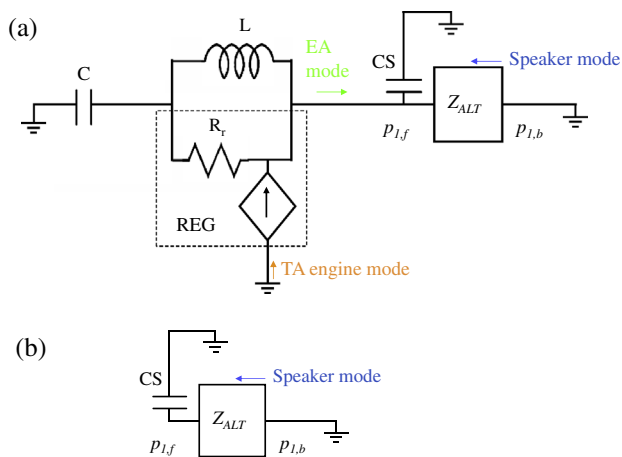


Fig. 2. TASE acoustic representation [35]. (a) Complete TA system and (b) alternator alone, used as dummy load.

acoustic pulse characteristics, especially the pressure amplitude, the volumetric flow rate and their respective phase differences, since the operation generally depends on specific coupling of the engine and its load.

- (1) In the *TA engine analysis mode* (orange), the thermoacoustic engine is driven as an engine converting heat into electrical power. It is coupled to an acoustic load represented by the linear alternator along with its electrical wiring and external load. This point of view is quite complex and typically numerical tools such as DeltaEC™ or Sage™ are preferred over the analytical perspective due to the challenges found in capturing the complete picture.
- (2) In the *speaker analysis mode* (blue) the linear alternator is driven by an external AC power supply to produce acoustic power. It has an acoustic load defined by the engine geometry and acoustic elements. In another form of the speaker mode the alternator is separated from the TA engine and its connections to the TA engine are blocked. We refer to

the alternator operated in this mode as “dummy” load. Using this point of view allows for the assessment of each acoustic building block.

- (3) In the *EA analysis mode* (green) there is an assumed acoustic pulse characterized by Δp_1 and U_1 and their phase difference $\phi_{\Delta p_1, U_1}$. In this point of view the electro-acoustic efficiency η_{EA} may be estimated. The alternator physical neighborhood in its pressure vessel is complex: The piston front face (in the compression space) is acted upon by the acoustic wave generated by the TASE loop. The moving mass is connected with springs to the housing vessel and separated from its cylinder with a small seal gap. The back space geometry is complex due to the inductor, springs bolts and fixtures, and effectively operates as a “gas spring” due to the gas volume compressibility.

Normally, when the system is operated in an *engine mode* the electrical impedance will include an external resistor as well as a coil with internal Ohmic resistance and an inductor, connected in series to an accompanying power factor-correcting capacitor, whereas working in a *speaker mode*, it should include an AC power source instead of the external capacitor and the resistive load.

Fig. 3 shows simplified schematics of a reciprocating piston inside its vessel attached to the TASE. The moving magnet having a mass m and piston with cross sectional area A is driven by the acoustic pulse, while being mechanically connected to a spring having a constant K , and electrically connected to an external load. The amplitude and phase of the acoustic pulse Δp_1 is simply the difference between the pressure oscillations on the front $p_{1,f}$ and the back side $p_{1,b}$.

$$\Delta p_1 = p_{1,f} - p_{1,b}. \quad (3.1)$$

Since the fundamental electro-acoustic transduction analysis is complex, it may be useful to begin the discussion with the most basic models of the acoustic portion and electrical portion alone, while later combining all elements into the full picture, and finally extending the model to include additional subtle phenomena.

Notice that the three different perspectives discussed in this section, regarding Fig. 2, are presented in the acoustic viewpoint, elaborated next.

3.1. Acoustic viewpoint

Considering the force balance on a single oscillating piston, a linear acoustic analysis is discussed next assuming all variables are not frequency-dependent. In this analysis an acoustic point of view is taken by considering a constant cross section – A which

is acted upon by the oscillatory force – $\Delta p_1 A$, where Δp_1 is the pressure difference across the piston. This pressure difference (force over the cross section) is imposed on the piston by the TA engine through the volumetric flow rate – U_1 and the local acoustic impedance Z_a

$$\Delta p_1 = -U_1 Z_a. \quad (3.2)$$

Once current begins to flow in the alternator coil having a length l , a Lorentz force – $\Delta p_{ALT} A$ will be formed due to the magnetic induction – B

$$\Delta p_{ALT} = \frac{I_1 B l}{A}. \quad (3.3)$$

Furthermore, there is also a pressure drop due to internal mechanical forces – Δp_{int} due to the piston's mass – m attached to a spring with constant – K , and having mechanical damping – R_m . The internal pressure drop term is best described by the mechanical impedance Z_m :

$$\Delta p_{int} = \frac{U_1}{A^2} Z_m = \frac{U_1}{A^2} \left(\frac{K}{i\omega} + i\omega m + R_m \right). \quad (3.4)$$

Combining Eqs. (3.2)–(3.4) results in an overall acoustic pressure difference balance described in the *EA analysis mode* as:

$$0 = \Delta p_1 + \Delta p_{ALT} + \Delta p_{int} = -Z_a U_1 + \frac{I_1 B l}{A} + Z_m \frac{U_1}{A^2} \quad (3.5)$$

Fig. 4 shows a diagram of the associated pressure differences described above.

3.2. Electrical viewpoint

Because of the acoustic oscillations of the alternator piston inside the coil, an electromotive force – V_{EMF} is produced due to the change in time of the magnetic flux

$$V_{EMF} = \frac{U_1 B l}{A}. \quad (3.6)$$

Often, (Bl) , the product of the magnetic induction B and an effective length l is used as an effective alternator constant.

Considering the coil as a perfect electrical inductor L_e having an electrical resistance R_e , a voltage V_{int} will develop between the coil terminals once a current I_1 flows through. This can be stated in terms of internal electrical impedance – $Z_{e,int}$, while an additional power factor correcting capacitor C_e may be added in series

$$V_{int} = -I_1 Z_{e,int} = -I_1 \left(R_e + i\omega L_e + \frac{1}{i\omega C_e} \right). \quad (3.7)$$

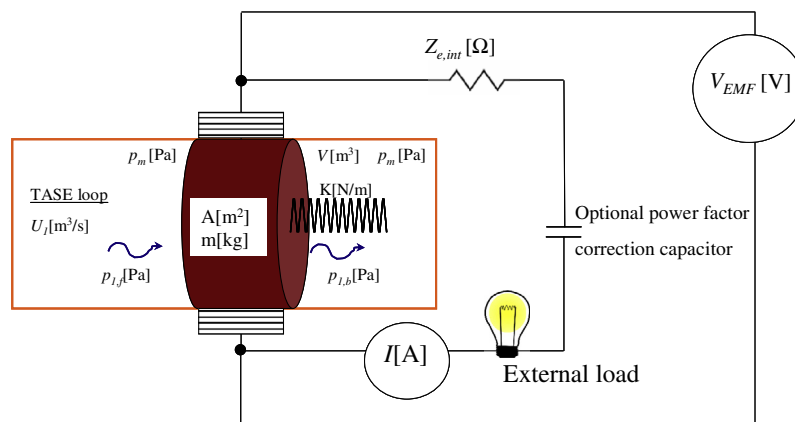


Fig. 3. Physical picture of the alternator, oscillating inside a pressure vessel, connected to an external electrical load.

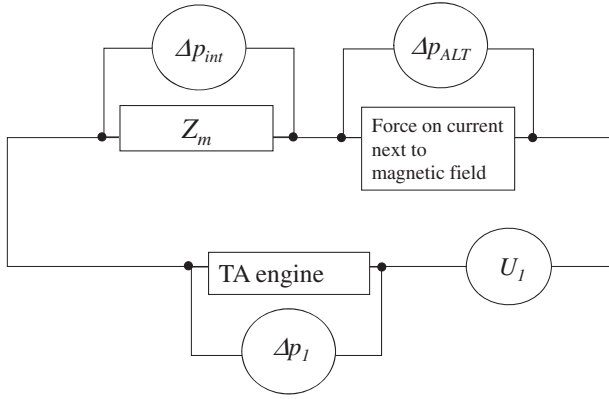


Fig. 4. Acoustic diagram, showing the pressure difference balance at the alternator-TA engine interface.

The useful voltage V_1 which can be derived from the system would be consumed by the resistive external load R_L

$$V_1 = I_1 R_L. \quad (3.8)$$

$$Z_{e,ext} = Z_{e,int} + R_L. \quad (3.9)$$

Combining Eqs. 3.6, 3.7 and 3.8 results in an overall electric potential difference picture, neglecting any inductive or capacitive external elements which may be added later in order to improve the power factor.

$$V_1 = V_{EMF} + V_{int} = \frac{U_1(Bl)}{A} - I_1 \left(R_e + i\omega L_e + \frac{1}{i\omega C_e} \right). \quad (3.10)$$

Fig. 5 shows the diagram of the voltage balance described above.

3.3. Inclusive perspective- Linear alternator as electro-mechanical transducer

The combined overall picture consisting of both acoustic and electrical parts is shown in Fig. 6

The electro-acoustic transduction efficiency η_{EA} depends on the ratio between the electrical output power – $\tilde{P}_e$ and the acoustic power – $\tilde{P}_a$ produced by the TA engine

$$\eta_{EA} = \left| \frac{\tilde{P}_e}{\tilde{P}_a} \right| \quad (3.11)$$

The (time averaged) acoustic energy carried by a sound wave is normally stated in terms of the acoustic intensity – I_a (energy per unit time and unit area) as [36]:

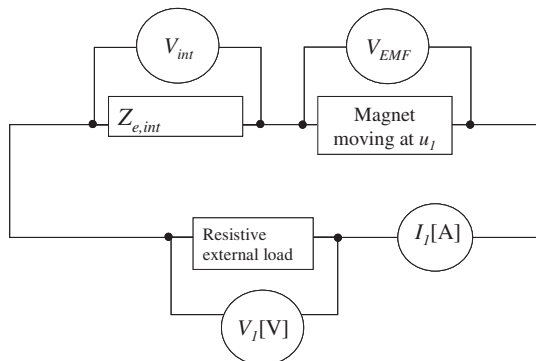


Fig. 5. Electric-potential impedance diagram for the alternator, and the accompanying electrical connections.

$$I_a = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \Re\{p_1 e^{i\omega t}\} \cdot \Re\{u_1 e^{i\omega t}\} dt = \frac{1}{2} \Re\{p_1 \cdot u_1^*\} \quad (3.12)$$

where u_1^* is the complex conjugate of the oscillatory part of the velocity, $\Re\{\}$ denotes taking the real part of a complex number, and p_1 is the oscillatory pressure amplitude. In terms of the acoustic power – $\tilde{P}_a$, the time-averaged energy flux flowing in the positive x direction per unit time (measured in Watts) at a specific location – x , through the normal cross section (y - z plane) – is in fact the space-averaged acoustic intensity:

$$\begin{aligned} \tilde{P}_a(x) &= \int_{\text{cross section}} I_a dA = \frac{1}{2} \Re\{p_1 \cdot U_1^*\} \\ &= \frac{1}{2} |p_1(x)| \cdot |U_1(x)| \cos(\phi_{p_1, U_1}) \end{aligned} \quad (3.13)$$

where ϕ_{p_1, U_1} is the (temporal) phase between the volumetric flow rate – U_1 and the oscillatory pressure p_1 .

Considering Eqs. (3.13) and (3.5) the time averaged acoustic power flowing in the TA engine compression space, from the acoustic loop into the alternator piston, is given by

$$\tilde{P}_a[W] = \frac{1}{2} \Re\{\Delta p_1 U_1^*\} = -\frac{1}{2} \left[\frac{(Bl)}{A} \Re\{U_1 I_1^*\} + \frac{R_m}{A^2} |U_1|^2 \right] \quad (3.14)$$

while the average electrical power flowing from the alternator into the external load is found by considering Eq. (3.10)

$$\tilde{P}_e[W] = \frac{1}{2} \Re\{V_1 I_1^*\} = \frac{1}{2} \left[\frac{(Bl)}{A} \Re\{U_1 I_1^*\} - R_e |I_1|^2 \right]. \quad (3.15)$$

Combining Eqs. (3.14) and (3.15) results in the following expression for the electro-acoustic efficiency, clearly demonstrating how the acoustic impedance affects the electro acoustic efficiency (see Appendix A.1):

$$\eta_{EA} = \frac{\Re\{Z_a\} - \frac{R_m}{A^2} - R_e \left(\frac{A}{Bl} \right)^2 \left| Z_a - \frac{Z_m}{A^2} \right|^2}{\Re\{Z_a\}}. \quad (3.16)$$

An alternative form for the electro-acoustic transduction efficiency η_{EA} Eq. (3.16), focusing on the effect of the electrical load R_L , results from the definition of the external electrical impedance $Z_{e,ext}$ (3.9) (Appendix A.2):

$$\eta_{EA} = \frac{R_L}{R_e + R_L + \frac{|Z_{e,ext}|^2 R_m}{(Bl)^2}}. \quad (3.17)$$

Although the wish for low friction R_m , minimum electrical resistance R_e as well as working at electrical resonance might seem obvious, indicating the best external load value R_L , as well as having large (Bl) might be essential for a specific application. Moreover, the critical effects of mechanical friction and internal electrical resistance on the electro-acoustic efficiency do not allow choosing the best external load value *a priori* [37]. Considering operating at electrical resonance ($Z_{e,ext} = R_e + R_{e,ext}$) with a given (Bl) value of 50 N/A, Fig. 7 shows the effect of different R_e values on η_{EA} at $R_m = 10$ Ns/m, while Fig. 8 shows the effect of different R_m [Ns/m] values on η_{EA} at $R_e = 1 \Omega$.

Knowing the exact values of mechanical friction and internal electrical resistance allows choosing the (theoretically) best R_L value in view of Eq. (3.17). However, indicating the exact R_m value may require some precaution, as will be discussed next.

Using the thoroughly explained practice of electro-mechanical transduction [38,39], the alternator's efficiency while operating in the *speaker analysis mode* can be found by the same kind of linear analysis of the overall acoustic-mechanical-electrical impedance of the experimental apparatus [40]. A compacted form for this point of view is shown in Fig. 9:

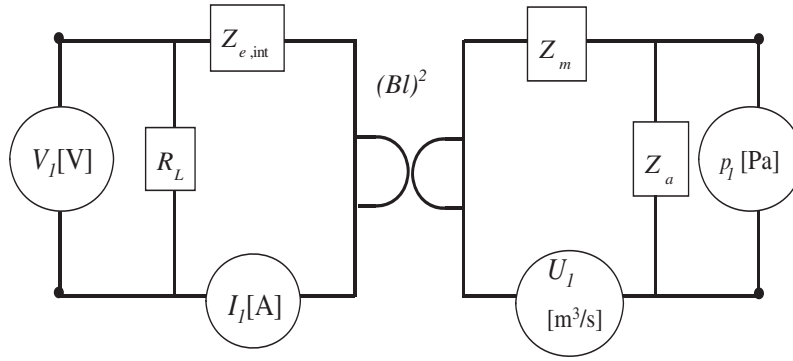
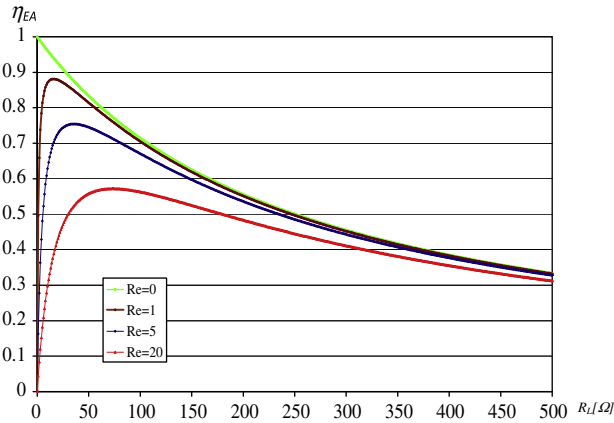
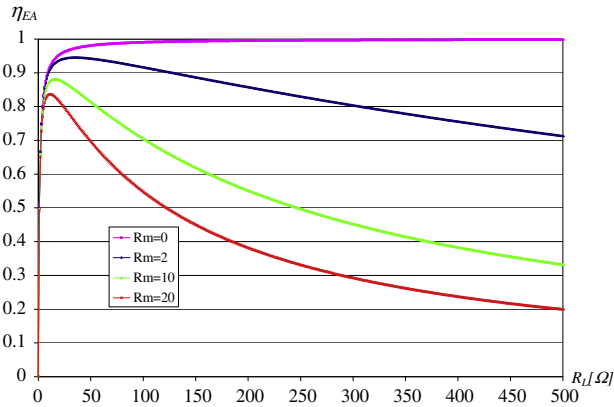


Fig. 6. Combined electro-acoustic transduction impedance diagram.

Fig. 7. Internal electrical resistance R_e effect on electroacoustic transduction efficiency η_{EA} vs. external load resistance R_L , at constant mechanical friction factor – $R_m = 10$ Ns/m.Fig. 8. Mechanical friction R_m effect on electroacoustic transduction efficiency – η_{EA} vs. external load resistance R_L , at constant electrical internal resistance $R_e = 1 \Omega$.

Notice the reciprocal definitions of electroacoustic transduction efficiency between the *engine mode* Eq. (3.11), and η_{AE} for the *speaker mode*

$$\eta_{AE} = \frac{\tilde{P}_a}{\tilde{P}_e} \quad (3.18)$$

Since the energy-flow changes direction while working in the *speaker mode* relative to the *TA engine mode*, the pressure balance described in (3.5) for the *TA engine* needs to be modified for the *speaker mode* according to

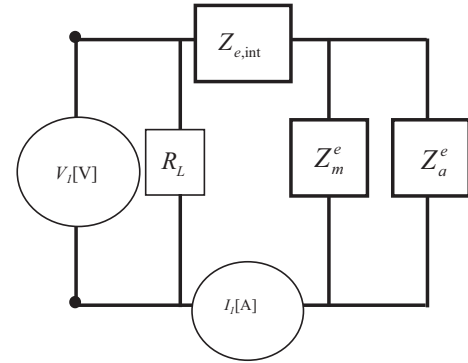


Fig. 9. Compact impedance diagram of electroacoustic transduction.

$$0 = \Delta p_1 + \Delta p_{ALT} - \Delta p_{int} \quad (3.19)$$

$$\frac{I_1(Bl)}{A} = U_1 \left(\frac{Z_m}{A^2} + Z_a \right).$$

In the latter case, the energy flow direction is from the linear alternator into the acoustic load, following some internal pressure drop as well as mechanical dissipation originating from the $Z_m U_1$ term in (3.4). In the following section the *speaker mode* will be used for evaluating the mechanical friction R_m from the experimentally measured amplitudes of the current and piston velocity.

Surely, there are various alternative forms for representing the overall impedance diagram in both acoustic framework or electrical framework [28,29,22,39,40]; however in this paper the following notation is used when considering Figs. 9 and 10:

Electrical equivalent of mechanical impedance

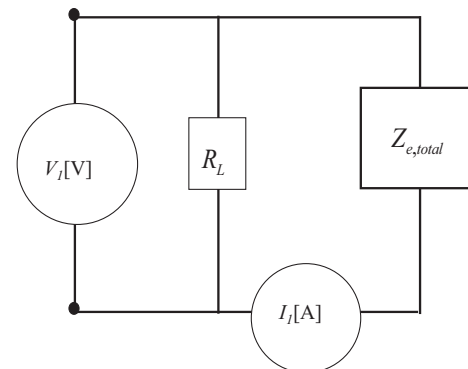


Fig. 10. Overall electrical representation.

$$Z_m^e = \frac{(Bl)^2}{Z_m}. \quad (3.20)$$

Electrical equivalent of acoustic impedance

$$Z_a^e = \frac{(Bl)^2}{A^2 Z_a}. \quad (3.21)$$

Internal electrical impedance

$$Z_{e,int} = R_e + i\omega L_e + \frac{1}{i\omega C_e} \quad (3.9a)$$

Overall impedance

$$Z_{e,total} = Z_{e,int} + \frac{1}{\frac{1}{Z_m^e} + \frac{1}{Z_a^e}} \\ = R_e + i\omega L_e + \frac{1}{i\omega C_e} + \frac{(Bl)^2}{R_m + i\omega m + \frac{k}{i\omega} + A^2 Z_a}. \quad (3.22)$$

Yet another simplification may be introduced by considering the mechanical–acoustical impedance $Z_{m,a}$

$$Z_{m,a} = Z_m + A^2 Z_a. \quad (3.23)$$

$$Z_{e,total} = Z_{e,int} + \frac{(Bl)^2}{Z_{m,a}}. \quad (3.24)$$

Considering the above mentioned shortcuts, the current in the loop I_1 is related to the externally measured (or supplied) voltage V_1 , Fig. 10.

$$V_1 = I_1 R_L. \quad (3.25)$$

$$I_1 R_L + I_1 Z_{e,total} = 0. \quad (3.26)$$

The complexity in achieving the highest overall efficiency in a thermoacoustic system by correct impedance matching has been discussed thoroughly by Wakeland [28], while several experimental verification techniques have been suggested by Liu and Garrett [37], Paek et al. [29] and Yu et al. [21,22]. Under most cases the optimal design is found through some compromise between maximum output power and overall efficiency. Moreover, usually there is some degree of freedom in the integration of a given alternator and a thermoacoustic engine. Recent publications disclose two noteworthy coupling methods of electro-acoustic coupling [7,10]. Furthermore, it can be found that the volumetric flow rate U_1 is related to the input voltage V_1 through (Appendix A.3):

$$U_1 = \frac{(Bl)AV_1}{Z_{e,int}(Z_m + A^2 Z_a) + (Bl)^2} \quad (3.27)$$

Based on these analytical considerations, proper selection of impedance values is crucial. Some degree of freedom remains available for optimization of a complete electricity-producing thermoacoustic-system, despite technological challenges such as minimizing friction, maintaining tight seal gaps and leaks prevention, as well as practical limitations related to piston stroke, mechanical springs elongation and dead volumes. However, accurate evaluation of critical impedance values is essential for comprehensive optimization.

4. Experimental setup and results

This section starts with a short description of a simple preliminary test procedure for evaluation of mechanical friction, followed by a general description of our experimental apparatus, concluding with detailed impedance assessment and validation of the analytical model.

During the experimental stage a custom design of STAR 1S102M/A linear alternator [41], Fig. 11, was used in several

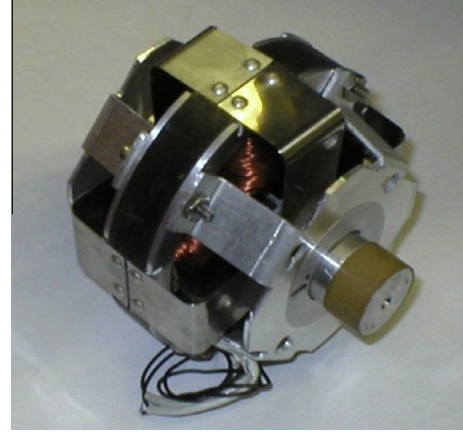


Fig. 11. Qdrive's 1S102M/A linear reciprocating alternator.

different environments. The STAR has a complex design, having non-axi-symmetric “interdigitating interfaces” between moving and fixed magnetic circuit elements [42]. Nevertheless, its key operational principles are common to all linear alternators, where a conversion of power occurs from an oscillatory mechanical energy into an electrical energy in the form of alternating current. Similar to common linear alternators [43], this device has a ½ kg piston reciprocating back and forth on a linear trajectory, inducing a change of magnetic flux in the surrounding inductor wire loop. The voltage across the loop is proportional to the rate of change of the magnetic flux through the wire loop according to Faraday's law.

4.1. Preliminary testing

The simplest approach for mechanical friction determination is by following the free decay of a linear alternator's movement [44], placed in standard atmospheric conditions and held firm by its stand. It would seem rational to implement this by giving the alternator a jolt, either mechanical or electrical, thus disturbing its equilibrium and causing the piston to oscillate at its natural frequency until brought back to rest by the friction. However, this is impractical when the alternator is mounted in its pressure vessel and the TASE filled with gas. Therefore, to apply the Resonance Decay method, the alternator's natural resonance frequency must be identified first, for example by locating the peak of a frequency response measurement, as described next in Section 4.2; while later the piston is driven at its resonance frequency using an external power supply, until the measurement process begins as the driving power source is shut abruptly. Fig. 12 shows the theoretical approximate displacement fit $\xi_{fit}(t)$ (red¹) from Eq. (4.1) [44], for the experimental damped harmonic oscillations (blue), indicating mechanical friction characterized by $R_m = 6.9$ [Ns m⁻¹], measured in the laboratory at 1 atmosphere of air at room temperature.

$$\xi_{fit}(t) = |\xi| \sin(\omega t) e^{-R_m t/2m}. \quad (4.1)$$

Note that this preliminary test method does not provide the full frequency dependence of friction. In a practical system it is necessary to determine the friction at the operating frequency, which may be different from the natural frequency. Yet, in our preliminary phase experiments, performed approximately one year after the alternator was purchased, the measured value was somewhat different than the manufacturer's specified value of $R_m = 5.5$ [Ns m⁻¹], which seems reasonable since some inevitable wear has meanwhile

¹ For interpretation of color in Figs. 7, 8, 12, and 14–17, the reader is referred to the web version of this article.

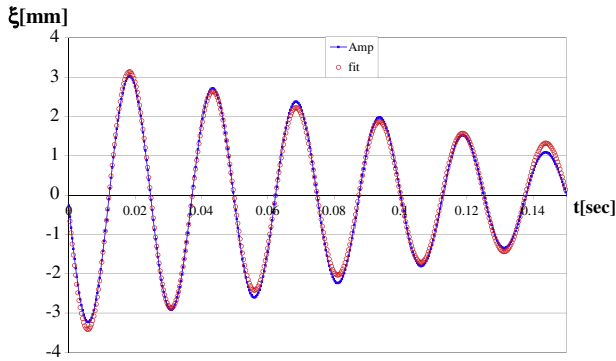


Fig. 12. Alternator piston's free decay at 39.8 Hz.

occurred.

4.2. Impedance assessment and validation testing

To confirm the validity of the complete analytical model described above, as well as assessing critical impedance values, a set of experiments was conducted. The experimental apparatus details are presented briefly, followed by a specific description of the laboratory setup, closing with highlights of the major results.

Our experimental system shown in Fig. 13 consisted of a TASE equipped with an alternator, as described in Fig. 1.

During the experimental testing phase, the system was driven both as an engine, Fig. 1(a), converting heat into electrical power, or as a speaker, producing acoustic power driven by an electrical AC power supply. For that purpose, an adaptable experimental system was fabricated, assembled, and operated at different conditions. The versatile system allowed for a good comparison between sequential sets of experimental testing. In an additional “dummy” speaker configuration, the compression space was blocked, thereby separating the alternator section from the TASE loop, hence modifying the compression space acoustic impedance, as well as detaching the regenerator and heat exchangers from the available acoustic power.

In the speaker mode, the system typically consisted of a dual-opposed linear alternators driven by an external programmable AC Source, Chroma model 61504 [45], attached to either a thermoacoustic Stirling engine, or to a simplified “dummy” load

volume. The entire experimental apparatus included two types of electricity generating ThermoAcoustic Stirling Engine models, termed TASE-200 and TASE-B. Each TASE may be operated as an engine – converting heat into electricity, or in a reverse mode of operation – employing electricity for heat pumping under a temperature difference at the desired working conditions. During the engine mode of operation, the three heat exchangers are used to supply and remove heat from the engine, as the thermoacoustic oscillations are driving the alternators producing electricity, consumed by an external electrical load. In the reverse mode of operation a pair of dual opposed linear alternators are powered by the external electrical power supply, producing acoustic power which may be used to pump heat from the low temperature to the high temperature heat exchanger. These models of thermoacoustic engines operate much like earlier versions described in the literature [4,5,15]. During the electro acoustic transduction testing, the alternators were connected in series to the external power supply, delivering the AC current at predetermined frequencies, as the external voltage across the alternators was kept constant throughout the frequency sweep.

Two types of pressure sensors were installed along the loop at critical points, comprising Endevco 8510B [46] piezoresistive pressure transducer, as well as a PCB Piezotronics [47] piezoelectric subminiature ICP® pressure sensor model 105C12, along with 20 thermocouples situated in the thermodynamic section, spanning the heat exchangers, regenerator and the thermal buffer tube walls. The piston's displacement was measured using LVDT Macro-sensors model CD-375-250 [48] while the current and voltage readings were fed directly into the NI compact RIO system [49]. All the required data acquisition and preliminary analysis were performed in real time using the Labview™ program. During each test, the alternator piston was excited at various driving frequencies as the supplied voltage remained constant. The calculated velocity amplitude found by measuring the displacement amplitude at each driving frequency, may be compared with the velocity u deduced from the analytical model (Eq. (4.2) derived from Eq. (3.27)) for a known piston area, as shown in Fig. 14.

$$u_1 = \frac{(Bl)V_1}{Z_{e,int}(Z_m + A^2Z_a) + (Bl)^2} \quad (4.2)$$

To calculate the velocity u_1 the following procedure was used: the density ρ_m was derived from the helium properties at fill pressure p_m of 30 bar at room temperature; the parameters K , m and (Bl)

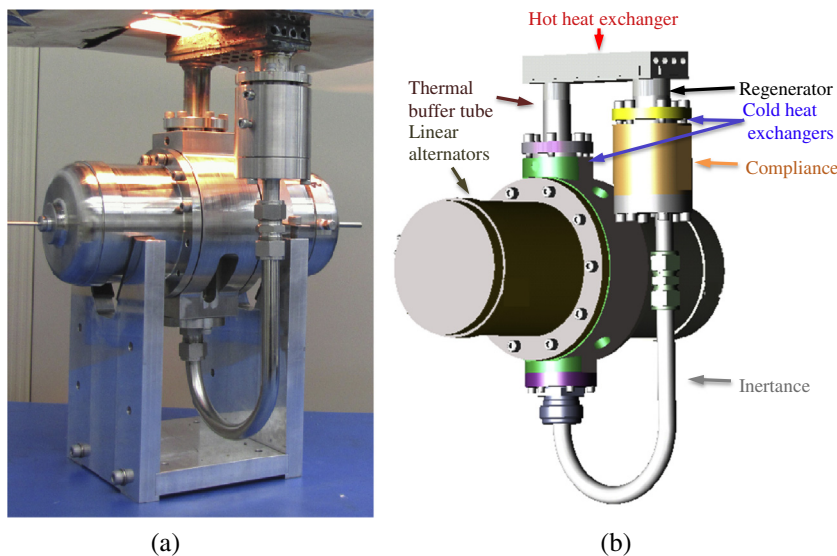


Fig. 13. Experimental setup of the tested system. (a) Actual TASE Photograph and (b) schematic system illustration.

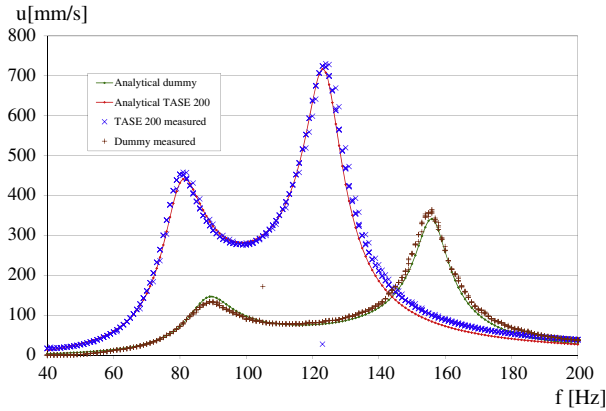


Fig. 14. Alternator's piston frequency response with 3 MPa helium gas.

were taken from the alternator's data sheet; the electrical values V_1 , R_e , L_e and C_e were directly measured using standard tools, and used to calculate $Z_{e,int}$ from (3.8). The acoustic portion Z_a required for Eq. (4.2) was evaluated considering Fig. 2, Eq. (2.12) and (2.13) using the geometrical dimensions of the apparatus, having a 2" diameter piston and compression space volume of $5\text{E}-5\text{m}^3$. The inertance had a cross section area of $1\text{E}-4\text{m}^2$, as its lengths for the dummy and TASE200 were $L_{dummy} = 0.14\text{ m}$, $L_{TASE200} = 0.51\text{ m}$ respectively, while the TASE200 included additional effective compliance volume $V_{TASE200} = 5.11\text{E}-5\text{m}^3$. To evaluate the mechanical friction factor R_m from this velocity–frequency curve we guessed various values of R_m and used them to evaluate Z_m from (3.4) until a good correlation between the calculated and the measured values of the velocity amplitude was obtained. The results presented in Fig. 14 qualitatively confirm the analytical model, though a slight discrepancy may be observed at high frequencies. The left peak is mainly contributed by the electrical portion, while the right peak is mainly dominated by the mechanical-acoustic portion.

Comparing the two techniques described above, the frequency response method and the alternator's free decay method, each has its own advantages. The free decay method is more focused on mechanical effects, while the frequency response method depends also on the acoustic impedance Z_a as well as on electrical parameters. Moreover, in the free decay method, the effect of frequency-dependent phenomena on the alternator's impedance is concealed. This may become misleading, for example in a thermoacoustic engine, where the working frequency is normally different from the alternator's natural frequency. The weakness of the suggested method for evaluating R_m by correlating the measured data with the velocity calculated from Eq. (4.1) originates from the requirement of using some alternative techniques in order to attain prior knowledge of the exact values of the electrical parameters – R_e , L_e , C_e .

Already during early testing stages, the frequency response method yielded a very high mechanical friction factor R_m , about three times larger than the free decay method. This discrepancy may originate from different oscillating flow patterns in the piston's vicinity due to dissimilar acoustic environments between the two experiments; therefore a more accurate method was sought. Notice the main difference in the experimental procedure, where in the open air free decay method the alternator was fixed in an open test bench, while during the frequency response set of experiments the alternator was surrounded by a helium atmosphere inside a pressure vessel at varying pressures. Nevertheless, the two configurations tested had the same alternators, but located inside different acoustic environments.

To resolve the inferred difference in friction factor, as well as further validate the analytical model effectiveness, the data was evaluated in light of another measurable quantity – the transfer impedance Z_c , defined here as (Eq. (3.20)):

$$Z_c \equiv \frac{|I|}{|u|} = \frac{|Z_{m,a}|}{|(Bl)|} \quad (4.3)$$

Since the transfer impedance is not affected by the electrical impedance, while enabling simple exploration of different working conditions, it may become more adequate for highlighting the acoustic-mechanical effects. In order to best estimate the friction factor at the working point of a thermoacoustic engine, it was found that the transfer impedance method was the preferred method among the three options discussed in this paper.

Several different acoustic environments were experimentally attainable, depending mostly on the gas free volume V , as well as the mean fill pressure p_m , forming together an almost adiabatic “gas spring” having an effective spring constant – $K_{gas-spring}$ acting on the piston face at a cross section area A_p :

$$K_{gas-spring} \approx \frac{\gamma p_m A_p^2}{V}. \quad (4.4)$$

By changing either the mean pressure while keeping the volume of the Dummy fixed at $V_{dummy} = 3\text{E}-5\text{ m}^3$ (Fig. 15), or by changing the free volume while keeping the 30 bar mean pressure (Fig. 16) – different peak locations are evident in the transfer impedance Z_c frequency response curve. These changes are expected considering an effective spring constant K_{eff} affecting the magnitude of $Z_{m,a}$ on the right hand side of Eq. (4.3):

$$K_{eff} \approx K + K_{gas-spring}. \quad (4.5)$$

Hence, the spring constant K used in Eq. (3.4) for Z_m should be replaced by K_{eff} according to Eq. (4.5).

The transfer impedance method described above allows for a straightforward comparison between the analytical model and the laboratory measured values, keeping the mechanical parameters constant, while allowing for several system configurations, over a broad frequency range as well as different fill pressures. Moreover, by measuring an engine's transfer impedance at room temperature one can predict the operating frequency before its usage in the field. This may become useful for fine tuning of external hardware such as balancer for vibration cancellation, capacitor and starter. Yet another possible benefit from the transfer impedance measurement is an estimation of an effective (Bl) product as well as a measurement of L_e at typical working conditions (I , ω , ξ), as will be demonstrated in a following paper.

The correlations between the analytically predicted values (solid lines) and the laboratory measured results (experimental

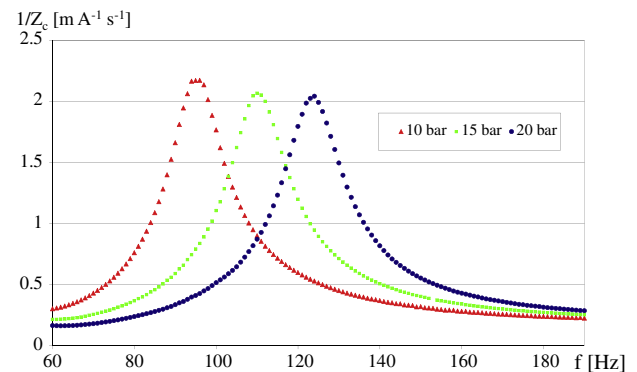


Fig. 15. Mean pressure effect on the measured transfer impedance Z_c , demonstrating different frequency curves due to altered stiffness (gas spring effect).

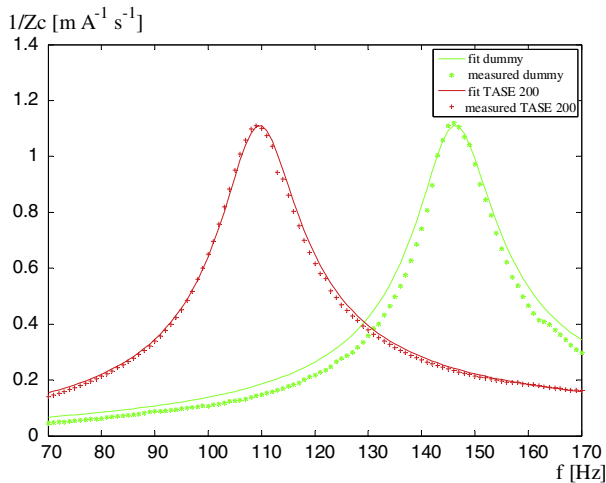


Fig. 16. Mutual fit (solid lines) for the measured transfer impedances Z_c of TASE 200 (+) and dummy load (*).

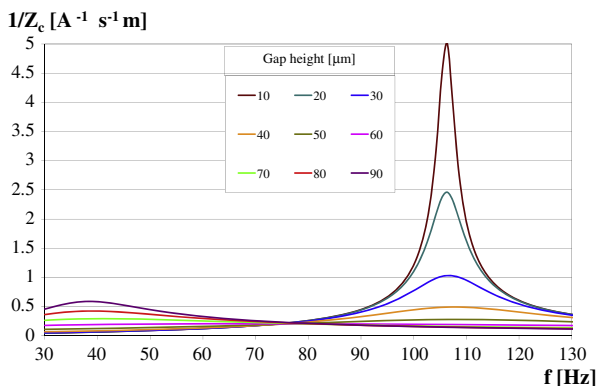


Fig. 17. Simulated effect of different gap heights (10–90 μm) on transfer impedance Z_c .

values) in Fig. 16 seem reasonable for both system configurations – TASE200 (+, red) and dummy (*, green). However, the mechanical friction inferred from the fit shown in Fig. 16, $R_m^* = 45$ [Ns/m], is higher by a factor of ~ 6 than the one determined in the natural free decay. Furthermore, the respective curve fitting for both configurations was unsatisfactory despite numerous parametric curve fitting trials, suggesting that a frequency dependent effect might be missing in the analytical model. It might be possible that viscous resistance as well as blow – by in the tight seal gap are the dominant frequency dependent phenomena to be considered.

This has initiated the process of extending the analytical model, to include the more subtle issues which may have been unnoticed before, using standard methods.

To account for the discrepancies, a numerical simulation using DeltaEC™ [50] was performed including viscous losses as well as blow – by in the seal gap. In the simulation even negligible effects such as thermal-relaxation dissipation losses [30] were considered. In the simulation, the externally supplied voltage V_1 is kept constant while the current, the pressure amplitude, and the piston velocity are allowed to vary as the frequency is changed. For each gap height a frequency sweep was carried out while forcing boundary conditions suitable for a 30 bar helium closed chamber. Fig. 17 shows the frequency response of the transfer impedance Z_c for five different gap heights. As the gap gets smaller the seal becomes tighter and the gas spring effect becomes more dominant, shifting

the resonance peak from its mechanical resonance at about 40–100 Hz. When the gap is open no pressure difference can remain between both piston sides, making the gas spring effect negligible, whereas in a tight configuration, the gas spring effect as well as viscous losses inside the gap become dominant.

Following a fine-tuning of the simulation to correlate with the experimental results the analytical model was extended to include the additional losses related with the seal gap. These results will be described in a following paper.

5. Summary and conclusions

One leading solution for a sustainable energy production is the traveling wave thermo acoustic Stirling heat engine; yet, its practical efficiency limitations seem to delay its market penetration. Despite its great promise as a highly reliable device, currently no such engine is available commercially, as the technology employed is not yet mature enough. It appears that one of the main obstacles for wide-ranging acceptance is the friction occurring in the only moving element – the linear alternator's piston flexure seals.

This research has provided a quantitative analytical examination of the influence of the critical physical parameters on the electro acoustic transduction efficiency η_{EA} , revealing the severity of frictional effects present in the system. A novel measurement method for precise determination of mechanical friction constant – R_m has been developed, indicating a mismatch in friction factor evaluation at the engine's working frequency, between the two methods described above – the free decay and the frequency response methods. Moreover, using a frequency independent R_m does not allow obtaining a satisfactory respective curve fitting for the experimental systems studied due to the involved complex interactions. Additionally, a direct measurement of an engine's transfer impedance at room temperature enables to find the exact natural frequency before field operation, and calibrate external hardware accordingly, leading to practical benefits.

In this study, two experimental systems (TASE200 and dummy) have been extensively explored at a variety of fill pressures to gain greater confidence in the correlations between the measurements and the analytical model. The two systems were tested repeatedly at a variety of frequencies and detailed measurements were conducted to evaluate the operating parameters. These experimental results, devised from a robust set of experimental testing, were used to calculate the relevant impedances. In addition, a detailed thermoacoustic simulation using DeltaEC™ was performed in order to verify the effect of the seal gap on the transfer impedance. Good agreement between the experiments and simulations was observed, while in a following paper we will extend the analytical model to include the effects of viscous friction in the tight seal gap, resulting in better match to the experimental values.

Public acceptance of products based on electricity generating thermoacoustic engines is expected to grow in the near term, as higher efficiencies are expected to be attained through proper implementation of specific elements, accessible through precise determination of the physical parameters present in the actual structure. Improved designs, based on the considerations explored in this paper, may lead to enhanced system performance, mostly in terms of the product overall cost per Watt, as well as the envisaged higher power levels.

Acknowledgement

The authors gratefully acknowledge the important information and insight gained in discussions with Dr. Steven L. Garrett.

Appendix A

A.1. Electroacoustic efficiency – acoustic impedance effect

Substituting the acoustical and electrical powers from (3.14) and (3.15), respectively, into the definition of the electroacoustic transduction efficiency (3.11) yields:

$$\eta_{EA} = \frac{|\tilde{P}_e|}{|\tilde{P}_a|} = \frac{\frac{(Bl)}{A} \Re\{U_1 I_1^*\} - R_e |I_1|^2}{\frac{(Bl)}{A} \Re\{U_1 I_1^*\} + \frac{R_m}{A^2} |U_1|^2} = \frac{\frac{(Bl)}{A} \Re\left\{\frac{I_1^*}{U_1}\right\} - R_e \frac{I_1^*}{U_1}}{\frac{(Bl)}{A} \Re\left\{\frac{I_1^*}{U_1}\right\} + \frac{R_m}{A^2}} \quad (A1.1)$$

From Eq. (3.5):

$$Z_a = (Bl) \frac{I_1}{AU_1} + \frac{Z_m}{A^2} \rightarrow \frac{(Bl)}{A} \frac{I_1^*}{U_1^*} = \left(Z_a - \frac{Z_m}{A^2}\right)^* \quad (A1.2)$$

and

$$\begin{aligned} R_e \frac{I_1}{U_1} \frac{I_1^*}{U_1^*} &= R_e \frac{A}{(Bl)} \left(Z_a - \frac{Z_m}{A^2}\right) \frac{A}{(Bl)} \left(Z_a - \frac{Z_m}{A^2}\right)^* \\ &= R_e \frac{A^2}{(Bl)^2} \left|Z_a - \frac{Z_m}{A^2}\right|^2 \end{aligned} \quad (A1.3)$$

Substituting in Eq. (A1.1):

$$\begin{aligned} \eta_{EA} &= \frac{\Re\left\{Z_a - \frac{Z_m}{A^2}\right\}^* - R_e \frac{A^2}{(Bl)^2} \left|Z_a - \frac{Z_m}{A^2}\right|^2}{\Re\left\{Z_a - \frac{Z_m}{A^2}\right\}^* + \frac{R_m}{A^2}} \\ &= \frac{\Re\{Z_a\} - \frac{\Re\{Z_m\}}{A^2} - R_e \frac{A^2}{(Bl)^2} \left|Z_a - \frac{Z_m}{A^2}\right|^2}{\Re\{Z_a\} - \frac{\Re\{Z_m\}}{A^2} + \frac{R_m}{A^2}} \end{aligned} \quad (A1.4)$$

and considering the expression for Z_m , Eq. (3.4):

$$\eta_{EA} = \frac{\Re\{Z_a\} - \frac{R_m}{A^2} - R_e \frac{A^2}{(Bl)^2} \left|Z_a - \frac{Z_m}{A^2}\right|^2}{\Re\{Z_a\}} \quad (A1.5)$$

A.2. Electroacoustic efficiency – engine mode

Considering Eqs. (3.10) and (3.25) while recalling the definition for the external electrical impedance $Z_{e,ext}$ (3.9)

$$\begin{aligned} V_1 &= \frac{U_1(Bl)}{A} - I_1 Z_{e,int} \rightarrow \frac{U_1(Bl)}{A} = I_1(R_L + Z_{e,int}) = I_1 Z_{e,ext} \\ V_1 &= I_1 R_L \end{aligned}$$

$$\frac{(Bl)}{A} \frac{I_1}{U_1} = \left(\frac{Bl}{A}\right)^2 \frac{1}{Z_{e,ext}} \quad (A2.1)$$

and substituting into the definition of the electroacoustic transduction efficiency according to Eq. (A1.1) yields:

$$\eta_{EA} = \frac{|\tilde{P}_e|}{|\tilde{P}_a|} = \frac{\frac{(Bl)}{A} \Re\left\{\frac{I_1}{U_1}\right\} - R_e \frac{|I_1|^2}{|U_1|^2}}{\frac{(Bl)}{A} \Re\left\{\frac{I_1}{U_1}\right\} + \frac{R_m}{A^2}} = \frac{\Re\left\{\frac{1}{Z_{e,ext}}\right\} - R_e \left|\frac{1}{Z_{e,ext}}\right|^2}{\Re\left\{\frac{1}{Z_{e,ext}}\right\} + \frac{R_m}{(Bl)^2}} \quad (A2.2)$$

Evaluating the complex terms

$$\Re\left\{\frac{1}{Z_{e,ext}}\right\} = \Re\left\{\frac{Z_{e,ext}^*}{Z_{e,ext} Z_{e,ext}^*}\right\} = \frac{\Re\{Z_{e,ext}\}}{|Z_{e,ext}|^2} = \frac{R_e + R_L}{|Z_{e,ext}|^2} \quad (A2.3)$$

and substituting back into Eq. (A2.1) yields:

$$\eta_{acoustic-electrical} = \frac{\frac{R_e + R_L}{|Z_{e,ext}|^2} - \frac{R_e}{|Z_{e,ext}|^2}}{\frac{R_e + R_L}{|Z_{e,ext}|^2} + \frac{R_m}{(Bl)^2}} = \frac{R_L}{R_e + R_L + \frac{|Z_{e,ext}|^2 R_m}{(Bl)^2}} \quad (A2.4)$$

A.3. Velocity calculation- speaker mode

Considering Eqs. (3.23)–(3.25) and (3.26)

$$\begin{aligned} V_1 &= I_1 R_L = -I_1 Z_{e,total} = -I_1 (Z_{e,int} + \frac{(Bl)^2}{Z_{m,a}}) \\ \Rightarrow I_1 &= -\frac{V_1}{Z_{e,int} + \frac{(Bl)^2}{Z_{m,a}}} \end{aligned} \quad (A3.1)$$

Recalling Eq. (3.10) and changing variables from the volumetric flow rate U_1 to velocity u_1 yields:

$$V_1 = V_{EMF} + V_{int} = u_1 (Bl) - I_1 Z_{e,int} \quad (A3.2)$$

Substituting Eq. (A3.1) into (A3.2):

$$\begin{aligned} (Bl)u_1 &= I_1 Z_{e,int} + V_1 = I_1 Z_{e,int} - I_1 (Z_{e,int} + \frac{(Bl)^2}{Z_{m,a}}) = -\frac{I_1 (Bl)^2}{Z_{m,a}} \\ \Rightarrow u_1 &= -\frac{I_1 (Bl)}{Z_{m,a}} \end{aligned} \quad (A3.3)$$

and concluding with the voltage V_1 from (A3.1):

$$u_1 = \frac{V_1}{Z_{e,int} + \frac{(Bl)^2}{Z_{m,a}}} \frac{(Bl)}{Z_{m,a}} = \frac{(Bl)V_1}{Z_{e,int} Z_{m,a} + (Bl)^2} \quad (A3.4)$$

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